Is the current metal removal rate formula for turning correct?

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Metal removal rate (MRR) in metalcutting indicates the volume of chips removed from the workpiece in one minute (the U.S. customary unit of measurement is in^3/min).

MRR is a measure of productivity: the higher the volume of chips removed, the higher the productivity of cutting. Therefore, the accuracy in calculating MRR is very important.

Currently, the MRR for turning is calculated by the formula:

$$Q = 12\ V_C \bullet a_p \bullet f \qquad (1)$$

Where V_C is a cutting speed (*sfm*), a_p is a depth of cut (*in.*), f is a feed rate (*ipr*), and 12 is a factor converting *inches* into *feet*.

The above formula is questionable, because it does not have a workpiece diameter. Depth of cut is always associated with the workpiece diameter. Before the turning process starts, the workpiece is uncut (diameter is D). When the turning process completes the first pass at a given depth of cut, the workpiece is cut into a smaller diameter (d).

The relationship between the uncut workpiece diameter (D), the cut workpiece diameter (d), and the depth of cut (a_p) is expressed by the formula:

$$a_p = (D - d)/2 \qquad (2)$$

Let's assume that the uncut workpiece (diameter D) is being turned at the depth of cut a_p. After the first pass, the cut workpiece diameter becomes:

$$d = D - 2a_p \qquad (3)$$

The imaginary gap between D and d is an annular ring, and its area (A) is calculated by the formula:

$$A = \pi\ (D^2/4 - d^2/4) = \pi/4\ (D^2 - d^2) \qquad (4)$$

Replacing d in formula (4) by its value from formula (3) results in the following equation:

$$A = \pi/4\ [D^2 - (D - 2a_p)^2] \qquad (5)$$

Simplification of equation (5):

$$A = \pi/4\ \{D^2 - [D^2 - 2(D \bullet 2a_p) + (2a_p)^2]\} = \pi/4\ (D^2 - D^2 + 4D \bullet a_p - 4a_p^2)$$

$$A = \pi/4\ [4\ (D \bullet a_p - a_p^2)]$$

$$A = \pi \bullet a_p \bullet (D - a_p) \qquad (6)$$

Volume of the annular cylinder or MRR is:

$$Q_1 = A \bullet L \qquad (7)$$

Where L is a distance traveled by the cutting tool in one minute a.k.a. feed speed. Feed speed is calculated by the formula:

$$L = f \bullet n \qquad (8)$$

Where n is a spindle speed:

$$n = 12\ V_c / \pi\ D \qquad (9)$$

Replacing L in formula (7) by its values from formula (8) results in the following formula:

$$Q_1 = A \bullet f \bullet n \qquad (10)$$

Replacing n in formula (10) by its values from formula (9) results in the following formula:

$$Q_1 = A \bullet f \bullet 12\ V_c / \pi\ D = 12\ V_c \bullet A \bullet f / \pi\ D \qquad (11)$$

Replacing A in formula (11) by its values from formula (6) results in the following formula:

$$Q_1 = [12\ V_c \bullet \pi \bullet a_p \bullet (D - a_p) \bullet f] / \pi\ D = [12\ V_c \bullet a_p \bullet (D - a_p) \bullet f] / D$$

$$Q_1 = [12\ V_c \bullet a_p \bullet (D - a_p) \bullet f] / D \qquad (12)$$

We performed numerous calculations and analyzed how the cutting parameters (workpiece diameter, depth of cut, feed rate, and cutting speed) affect the MRR.

As an example, the calculations were performed at the following constant parameters: depth of cut, feed rate, and cutting speed:

Depth of cut, $a_p = 0.200$ in.
Feed rate, $f = 0.020$ ipr
Cutting speed, $V_c = 500$ sfm

Workpiece diameters (D) were: 1.0, 2.0, 3.0, 4.0, and 5.0 inches. Metal removal rate (Q) was calculated by the currently used formula (1). Related parameters such as length of cut (L) and spindle speed (n) were calculated for each of the selected workpiece diameters. The results of the calculations are shown in Table 1.

Table 1 Metal removal rate and related parameters

Workpiece diameter, D in.	Metal removal rate, Q in^3/min	Spindle speed, n rpm	Length of cut, L in/min
1.0	24.0	1910	38.20
2.0	24.0	955	19.10
3.0	24.0	637	12.73
4.0	24.0	477	9.55
5.0	24.0	382	7.64

The area of annular ring (A), length of the cut (L), spindle speed (n), and metal removal rate (Q_1) were calculated by formulas (6), (8), (9), and (12) accordingly (Table 2).

Table 2 Metal removal rates and related parameters

Workpiece diameter, D in.	Metal removal rate, Q_1 in^3/min	Area of the annular ring, A in^2	Spindle speed, n rpm	Length of cut, L in/min
1.0	19.2	0.503	1910	38.20
2.0	21.6	1.131	955	19.10
3.0	22.4	1.759	637	12.73
4.0	22.8	2.388	477	9.55
5.0	23.0	3.016	382	7.64

As indicated by the above tables, the difference between Q and Q_1 decreases when the diameter of the workpiece increases. The MRR values calculated by formulas (1) exceed those calculated by formula (12): 25, 11, 7, 5, and 4% in accordance with selected workpiece diameters.

Conclusion

Accurately calculated MRR is very important, since the required machining power (P_m) depends on MRR and power constant (K_p) measured in $hp/in^3/min$

Required machining power is calculated by the following formula:

$$P_m = MRR \cdot K_p \ (hp)$$

Example

Workpiece: alloy steel AISI 4340, D = 1.0 in, hardness 200 HB, K_p = 0.72* $hp/in^3/min$, V_c = 500 sfm, a_p = 0.200 in, and f = 0.020 ipr.

Calculated:
Q = 24 in^3/min by formula (1) and $P_m = 24 \cdot 0.72 = 17.28$ hp
Q_1 = 19.2 in^3/min by formula (12) and $P_m = 19.2 \cdot 0.72 = 13.82$ hp

Assume that maximum rating power available from a given machine tool is 18 hp.
Since Q_1 = 19.2 in^3/min, the cutting speed can be increased by 25% (17.28/13.82 = 1.25), then $V_c = 500 \cdot 1.25 = 625$ sfm. Recalculated MRR by formula (12) is:

$$Q_1 = [12 \cdot 625 \cdot 0.2 \cdot (1.0 - 0.2) \cdot 0.02]/1.0 = 24 \ in^3/min$$

*Isakov, *Edmund. Cutting Data for Turning of Steel*, p.179, Table 4.2, Industrial Press, Inc., 2009, New York, NY 10018